Local correlations of mixed two-qubit states

Fu-Lin Zhang,[1] Chang-Liang Ren,[2,3] Ming-Jun Shi,[3] and Jing-Ling Chen[1,*]

[1] *Theoretical Physics Division, Chern Institute of Mathematics, Nankai University, Tianjin, 300071, P.R.China*
[2] *Department of Physics, Korea University, Seoul 136-713, Korea*
[3] *Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, P.R.China*

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The quantum probability distribution arising from single-copy von Neumann measurements on an arbitrary two-qubit state is decomposed to the local and nonlocal parts, in the approach of Elitzur, Popescu and Rohrlich[A. Elitzur, S. Popescu, and D. Rohrlich, Phys. Lett. A 162, 25 (1992)]. A lower bound of the local weight is proved being connected with the concurrence of the state $p_L^{\max} = 1 - \mathcal{C}(\rho)$. The local probability distributions for two families of mixed states are constructed independently, which accord with the lower bound.

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I. INTRODUCTION

Entanglement and nonlocality are two fundamental concepts in quantum description of nature, which are closely interconnected but not identical [1, 2, 3]. The former depicts the non-factorizability of the state of a composite quantum system [1]. And the latter is characterized by violation of a Bell inequality [2], which means the local measurement outcomes of the state cannot be described by a local hidden variables (LHV) model. For the case of pure states, Bell inequality violation is a witness of quantum entanglement [4]. But Werner [3] has shown a family of mixed entangled states (called Werner states now) can be described by a LHV model. The two concepts are not only the fundamental features of quantum theory, but also the crucial resources in quantum information [5, 6, 7, 8].

To quantify the degree to which a state is entangled, several measures have been proposed, such as entanglement of formation [9, 10, 11], entanglement of distillation [12], relative entropy of entanglement [13], negativity [14, 15], and so on. For two-qubit systems, the entanglement of formation is equivalent to a computable quantity, which is referred to as *concurrence* [10, 11]. The concurrence of a pure two-qubit state $|\psi\rangle = c_1|00\rangle + c_2|01\rangle + c_3|10\rangle + c_4|11\rangle$ is given by

$$\mathcal{C}(|\psi\rangle) = 2|c_1 c_4 - c_2 c_3|. \tag{1}$$

The pure state is equivalent to

$$|\psi(\theta)\rangle = \cos\theta|00\rangle + \sin\theta|11\rangle, \quad \theta \in [0, \pi/4], \tag{2}$$

under local unitary (LU) transformations [5], with concurrence $\mathcal{C}(|\psi(\theta)\rangle) = 2\cos\theta\sin\theta = \sin 2\theta$. For a mixed state, the concurrence is defined as the average concurrence of the pure states of the decomposition, minimized over all decompositions of $\rho = \sum_j p_j |\psi_j\rangle\langle\psi_j|$,

$$\mathcal{C}(\rho) = \min \sum_j p_j \mathcal{C}(|\psi_j\rangle). \tag{3}$$

It can be expressed explicitly as [10, 11]

$$\mathcal{C}(\rho) = \max\left\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right\}, \tag{4}$$

in which $\lambda_1, ..., \lambda_4$ are the eigenvalues of the operator $R = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ in decreasing order and σ_y is the second Pauli matrix.

The correlation in a bipartite quantum system is characterized by the probability distribution $P_Q(\alpha, \beta|\mathbf{a}, \mathbf{b})$ of the outcomes α and β, corresponding to the measurements labeled by $\mathbf{a}$ and $\mathbf{b}$ on the two subsystems respectively. It is called *local*, if the probability distribution can be simulated by a LHV model. Namely, there exists a shared classical variable λ distributed with probability measure μ such that

$$P_Q(\alpha, \beta|\mathbf{a}, \mathbf{b}) = \int d\mu(\lambda) P(\alpha|\mathbf{a}, \lambda) P(\beta|\mathbf{b}, \lambda), \tag{5}$$

where $P(\alpha|\mathbf{a}, \lambda)$ and $P(\beta|\mathbf{b}, \lambda)$ are the local response functions of the two observers. The form of the distribution in Eq. (5) leads to a set of constraints (Bell-type inequalities), which a local correlation should fulfill, for any fixed number of measurements on each subsystem. Therefore, Bell inequality violation is a sufficient condition of *quantum nonlocality*.

Elitzur, Popescu, and Rohrlich (EPR2) [16] use a different viewpoint to discuss the local and nonlocal contents of nonlocal probability distributions [see Eq. (6)]. Actually, EPR2 approach can be abstractly interpreted to answer such a question: whether an alternative description of nature is valid. Since the original work of EPR2 appeared, few papers generalized it in-depth. Recently, as the approach related to a more noticeable question, the simulation of quantum correlations with other resource, it attracted someone's attention again. Barrett

*Email:chenjl@nankai.edu.cn

et. al gave an upper bound of the weight of local component in $d \times d$ system [17]. In his recent work [18], Scarani reviewed the previous results and decomposed the quantum correlation P_Q corresponding to von Neumann measurements performed on the pure state (2) into a mixture of a local correlation P_L and a nonlocal correlation P_{NL}

$$P_Q = p_L(\theta)P_L + [1 - p_L(\theta)]P_{NL}, \tag{6}$$

in EPR2 approach. Scarani's construction of the local probability distribution P_L leads to

$$p_L(\theta) = 1 - \sin 2\theta, \tag{7}$$

which is an improved lower bound of $p_L^{\max}(\theta)$ on the original result $p_L^{\max}(\theta) \geq (1 - \sin 2\theta)/4$ given by EPR2. Here, $p_L^{\max}(\theta)$ denotes the maximum weight of the local component in Eq. (6). Further more, he presented an upper bound for $p_L^{\max}(\theta)$ on the family of pure two-qubit states and the first example of a lower bound on the local content of pure two-qutrit states.

It is interesting to note that the proportion of nonlocal correlation P_{NL} in Scarani's construction is nothing but the concurrence of $|\psi(\theta)\rangle$, $1 - p_L(\theta) = \sin 2\theta$. The main aim of this paper is to show this result can be generalized straightway to the mixed states case. Namely, we present a construction of P_L for arbitrary states ρ of two qubits, corresponding to the local weight $p_L(\rho) = 1 - \mathcal{C}(\rho)$. The construction will be proved as a theorem in Sec. II. In addition, we will give the EPR2 decompositions of some typical states in quantum information, such as the Generalized Werner state [19] and the mixture of a Bell state and a mixed diagonal state, of which Werner state and maximally entangled mixed states ρ_{MEMS} [20] are two spacial cases. Conclusion will be made in the last section.

II. EPR2 DECOMPOSITIONS OF MIXED TWO-QUBIT STATES

A. General Results

The probality that the local von Neumann measurements labeled by unit vectors $\mathbf{a}$ and $\mathbf{b}$ performed on the two qubits with state ρ lead to the outcomes (α, β) is

$$P_Q(\alpha, \beta|\rho; \mathbf{a}, \mathbf{b}) = \mathrm{Tr}(\Pi_A \otimes \Pi_B \rho), \tag{8}$$

with $\alpha, \beta = \pm 1$. Here, the projectors are given by

$$\begin{aligned} \Pi_A &= \frac{1}{2}(\mathbf{1} + \vec{\sigma} \cdot \mathbf{A}), \\ \Pi_B &= \frac{1}{2}(\mathbf{1} + \vec{\sigma} \cdot \mathbf{B}), \end{aligned} \tag{9}$$

where $\mathbf{1}$ is the 2×2 unit matrix, $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices in vector notation, and $\mathbf{A} = \alpha\mathbf{a}$ and $\mathbf{B} = \alpha\mathbf{b}$ are unit vectors. Then, the quantum probability distribution of the pure state $|\psi(\theta)\rangle$ can be obtained easily

$$P_Q(\theta) = \frac{1}{4}[1 + c(A_z + B_z) + A_z B_z + s(A_x B_x - A_y B_y)], \tag{10}$$

where $c = \cos 2\theta$ and $s = \sin 2\theta$ as denoted in [18]. Scarani improved the local probability distribution on EPR2's original construction to

$$P_L = \frac{1}{4}[1 + f(A_z)][1 + f(B_z)], \tag{11}$$

with the function $f(x) = \mathrm{sgn}(x) \min(1, \frac{c}{1-s}|x|)$. This keeps the product form in [16] and leads to $p_L = 1 - s = 1 - \mathcal{C}(|\psi(\theta)\rangle)$.

Whereas, the product form construction of P_L is obviously not optimal for mixed states because of the presence of classical correlation. A simple example is the separable state $\rho_s = (|00\rangle\langle 00| + |11\rangle\langle 11|)/2$, for which $P_Q(\rho_s) = (1 + A_z B_z)/4$ should be local completely. One can easily find the following equation is self-contradictory,

$$P_Q(\rho_s) = \frac{1}{4}(1 + f_A)(1 + f_B), \quad f_A, f_B \in [-1, 1], \tag{12}$$

if f_A and f_B are requested to be odd functions of $\mathbf{A}$ and $\mathbf{B}$ respectively. Actually, a straightforward construction of the local correlation of ρ_s is

$$\begin{aligned} P_L(\rho_s) &= \frac{1}{2}F^+(A_z)F^+(B_z) + \frac{1}{2}F^-(A_z)F^-(B_z) \\ &= P_Q(\rho_s), \end{aligned} \tag{13}$$

with $F^{\pm}(x) = \frac{1}{2}(1 \pm x)$. It contains a two-outcomes random variable with equiprobability as the LHV. The following results will show the local weight of an arbitrary two-qubit state satisfies $p_L = 1 - \mathcal{C}(\rho)$, if we choose the local probability distribution with a discrete LHV as

$$P_L = \sum_i \mu_i p_i(\mathbf{A}) q_i(\mathbf{B}), \tag{14}$$

where $\mu_i, p_i(\mathbf{A}), q_i(\mathbf{B}) \in [0, 1]$ are probabilities satisfying $\sum_i \mu_i = 1$, $p_i(\mathbf{A}) + p_i(-\mathbf{A}) = 1$ and $q_i(\mathbf{B}) + q_i(-\mathbf{B}) = 1$.

Theorem 1. The local content of the probability distribution for a two-qubit state ρ has a lower bound $p_L^{\max}(\rho) \geq 1 - \mathcal{C}(\rho)$.

Proof. Suppose $\{|\phi_i\rangle\}$ is a decomposition minimizing the average concurrence in Eq. (3), $\rho = \sum_i t_i |\phi_i\rangle\langle\phi_i|$, where $\sum_i t_i = 1$. All the elements in $\{|\phi_i\rangle\}$ are equivalent under LU transformation to the same state in the form of Eq. (2)

$$|\phi_i\rangle = U_i^A \otimes U_i^B |\psi(\theta)\rangle, \tag{15}$$

with the concurrence $\mathcal{C}(\rho) = \mathcal{C}(|\psi(\theta)\rangle)$.

Denote the unit vectors as $\mathbf{A}^{(i)}$ and $\mathbf{B}^{(i)}$ which satisfy $\vec{\sigma} \cdot \mathbf{A}^{(i)} = U_i^{A\dagger} \vec{\sigma} \cdot \mathbf{A} U_i^A$ and $\vec{\sigma} \cdot \mathbf{B}^{(i)} = U_i^{B\dagger} \vec{\sigma} \cdot \mathbf{B} U_i^B$. The quantum probability distribution is straightforward

to obtain

$$P_Q(\rho) = \sum_i t_i P_Q^{(i)}, \tag{16}$$

where $P_Q^{(i)} = \langle\psi(\theta)|\Pi_A^{(i)} \otimes \Pi_B^{(i)}|\psi(\theta)\rangle$ with $\Pi_A^{(i)} = \frac{1}{2}[\mathbf{1} + \vec{\sigma}\cdot\mathbf{A}^{(i)}]$ and $\Pi_B^{(i)} = \frac{1}{2}[\mathbf{1} + \vec{\sigma}\cdot\mathbf{B}^{(i)}]$. Each $P_Q^{(i)}$ can be decomposed in Scarani's approach as

$$P_Q^{(i)} = [1 - \mathcal{C}(\rho)]P_L^{(i)} + \mathcal{C}(\rho)P_{NL}^{(i)}, \tag{17}$$

where $P_L^{(i)}$ is defined in the form of Eq. (11) with $\mathbf{A}^{(i)}$ being substituted for $\mathbf{A}$ and $\mathbf{B}^{(i)}$ for $\mathbf{B}$. A naturally construction of the local probability distribution is $P_L(\rho) = \sum_i t_i P_L^{(i)}$, taking the form in Eq. (14). Then, one can obtain

$$P_Q(\rho) = [1 - \mathcal{C}(\rho)]P_L(\rho) + \mathcal{C}(\rho)\sum_i t_i P_{NL}^{(i)}, \tag{18}$$

which ends the proof. □

Since the procedure given by Wootters [11] to derive the optimal decomposition in Eq. (3) is effective but not easy to implement, we give the EPR2 decompositions of two families of typical mixed states in the following parts of this section. These are constructed directly, independent of the process presented above.

B. Werner State & Generalized Werner State

The Werner state [3] takes the form as

$$\rho_W = x|\psi^+\rangle\langle\psi^+| + (1-x)\frac{\mathbf{1}\otimes\mathbf{1}}{4}, \quad x \in [0,1], \tag{19}$$

where $|\psi^+\rangle = [|00\rangle + |11\rangle]/\sqrt{2}$ is one of the Bell basis. The concurrence $\mathcal{C}(\rho_W) = \max\{0, (3x-1)/2\}$. And its quantum probability distribution is given by

$$P_Q(\rho_W) = \frac{1}{4}[1 + x(A_zB_z + A_xB_x - A_yB_y)]. \tag{20}$$

When $x = 1/3$, ρ_W is separable, and $P_Q(\rho_W)$ can be represented as a local form

$$\begin{aligned} P_L^{1/3}(\rho_W) &= \frac{1}{6}[F^+(A_z)F^+(B_z) + F^-(A_z)F^-(B_z) \\ &\quad + F^+(A_x)F^+(B_x) + F^-(A_x)F^-(B_x) \\ &\quad + F^+(A_y)F^-(B_y) + F^-(A_y)F^+(B_y)] \\ &= P_Q(\rho_W)|_{x=1/3}. \end{aligned} \tag{21}$$

If we define the local distribution as

$$P_L(\rho_W) = \begin{cases} P_L^{1/3}(\rho_W), & x \geq 1/3\,; \\ 3xP_L^{1/3}(\rho_W) + (1-3x)\frac{1}{4}, & x < 1/3\,; \end{cases} \tag{22}$$

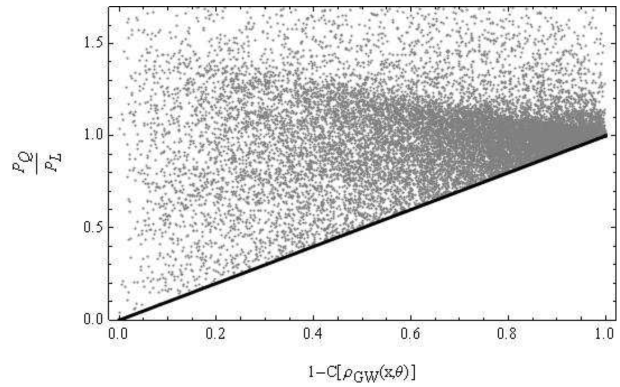

FIG. 1: Plot of 20000 randomly generated sets of entangled ρ_{GW} and unit vectors $(\mathbf{A},\mathbf{B})$ in the plane of $P_Q(\rho_{GW})/P_L(\rho_{GW}) \sim 1 - \mathcal{C}(\rho_{GW})$ in company with the line of $P_Q(\rho_{GW})/P_L(\rho_{GW}) = 1 - \mathcal{C}(\rho_{GW})$.

it is easy to prove $P_Q(\rho_W) = P_L(\rho_W)$ for $x < 1/3$ and $P_Q(\rho_W)/P_L(\rho_W) \geq \frac{3}{2}(1-x)$ for $x \geq 1/3$. The minimum of the radio occurs when the unit vectors $\mathbf{A}\cdot\mathbf{B'} = -1$ with $\mathbf{B'} = (B_x, -B_y, B_z)$. This indicates the local content of Werner state $p_L(\rho_W) = 1 - \mathcal{C}(\rho_W)$ corresponding to the construction of $P_L(\rho_W)$ in Eq. (22).

A family of generalized Werner state [19] is given by

$$\rho_{GW} = x|\psi(\theta)\rangle\langle\psi(\theta)| + (1-x)\frac{\mathbf{1}\otimes\mathbf{1}}{4}, \tag{23}$$

which is the mixture of the pure state (2) with the completely random state. Its concurrence is $\mathcal{C}(\rho_{GW}) = \max\{0, [(1+2s)x - 1]/2\}$, and quantum correlation can be obtained

$$\begin{aligned} &P_Q(\rho_{GW}) \\ &= \frac{1}{4}\{1 + x[cA_z + cB_z + A_zB_z + s(A_xB_x - A_yB_y)]\}, \end{aligned} \tag{24}$$

with s and c taking the definition in Eq. (10). As the treatment of Werner state, we start from the critical value of $x_c = 1/(1+2s)$, for which Eq. (24) is local obviously

$$\begin{aligned} P_L^{x_c}(\rho_{GW}) &= \frac{x_c}{2}\{c_+F^+(A_z)F^+(B_z) + c_-F^-(A_z)F^-(B_z) \\ &\quad + sF^+(A_x)F^+(B_x) + sF^-(A_x)F^-(B_x) \\ &\quad + sF^+(A_y)F^-(B_y) + sF^-(A_y)F^+(B_y)\} \\ &= P_Q(\rho_{GW})|_{x=x_c}, \end{aligned} \tag{25}$$

where $c_\pm = 1 \pm c$. When $x < x_c$, one can choose

$$\begin{aligned} P_L(\rho_{GW}) &= (1+2s)xP_L^{x_c}(\rho_{GW}) + [1-(1+2s)x]\frac{1}{4} \\ &= P_Q(\rho_{GW}) \end{aligned} \tag{26}$$

For the entangled region $x > x_c$, an appropriate construction of local distribution is given by the linear com-

bination

$$P_L(\rho_{GW}) = kP_L + (1-k)P_L^{x_c}(\rho_{GW}), \quad (27)$$

where P_L is the construction for pure state in Eq. (11), and $k = \frac{(1-s)[(1+2s)x-1]}{s[3-(1+2s)x]} \in [0,1]$ which is derived from the equation

$$\frac{c}{1-[(1+2s)x-1]/2} = k\frac{c}{1-s} + (1-k)\frac{c}{1+2s}. \quad (28)$$

Without analytical proof, we conclude from our numerical results that the construction (27) satisfies $P_Q(\rho_{GW})/P_L(\rho_{GW}) \geq 1 - \mathcal{C}(\rho_{GW})$, which is shown distinctly in Fig. 1. Hence, the local probability distributions in Eqs. (26) and (27) are corresponding to the content $p_L(\rho_{GW}) = 1 - \mathcal{C}(\rho_{GW})$.

C. Mixture of a Bell State and a Diagonal State

Another generalization of Werner state is the Bell State $|\psi^+\rangle$ mixed with a diagonal state

$$\rho_{BD} = \begin{bmatrix} x+\gamma/2 & 0 & 0 & \gamma/2 \\ 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ \gamma/2 & 0 & 0 & y+\gamma/2 \end{bmatrix}, \quad (29)$$

where the non-negative real parameters $x+y+a+b+\gamma = 1$. It contains many special two-qubit states, such as maximally entangled mixed states ρ_{MEMS} [20], frontier states of the bounds for concurrence [21] and so on. Its concurrence is $\mathcal{C}(\rho_{BD}) = \max\{0, \gamma - 2\sqrt{ab}\}$, which is independent on x and y. Our construction of the EPR2 decomposition of ρ_{BD} is divided into two steps: (i) We give the results of the spacial case of $x = y = 0$; (ii) The local distribution of the general case can be derived immediately base on the results of the first step.

(i) When $x = y = 0$, $\rho_{BD}^0 = \gamma|\psi^+\rangle\langle\psi^+| + a|01\rangle\langle 01| + b|10\rangle\langle 10|$, and the quantum probability distribution is

$$P_Q(\rho_{BD}^0) = \frac{1}{4}[1 + (a-b)(A_z - B_z) + (\gamma - a - b)A_zB_z + \gamma(A_xB_x - A_yB_y)], \quad (30)$$

where we choose $a \geq b$ without loss of generality. At the critical point of separability $\gamma = 2\sqrt{ab}$,

$$\begin{aligned} P_L^c(\rho_{BD}^0) &= \frac{1}{4}[F_A^+(A_x)F_B^+(B_x) + F_A^-(A_x)F_B^-(B_x) \\ &\quad + F_A^+(A_y)F_B^-(B_y) + F_A^-(A_y)F_B^+(B_y)] \\ &= P_Q(\rho_{BD}^0)|_{\gamma=2\sqrt{ab}} \end{aligned} \quad (31)$$

where the local response functions $F_A^\pm(x) = \frac{1}{2}(1 + \sin\vartheta A_z \pm \cos\vartheta x)$ and $F_B^\pm(x) = \frac{1}{2}(1 - \sin\vartheta B_z \pm \cos\vartheta x)$, with $\vartheta = \vartheta_c = \arcsin(\sqrt{a} - \sqrt{b})$. In the region $\gamma < 2\sqrt{ab}$, we assume

$$\begin{aligned} P_L(\rho_{BD}^0) &= gP_L^c(\rho_{BD}^0) + (1-g)P_L^0(\rho_{BD}^0), \quad (32) \\ P_L^0(\rho_{BD}^0) &= \lambda_+F^+(A_z)F^-(B_z) + \lambda_-F^-(A_z)F^+(B_z), \end{aligned}$$

where $\lambda_\pm = \frac{1}{2}(1 \pm \Delta)$ and $P_L^0(\rho_{BD}^0)$ takes the same form as $P_Q(\rho_{BD}^0)|_{\gamma=0}$. To hold the relation $P_L(\rho_{BD}^0) = P_Q(\rho_{BD}^0)$ when $\gamma < 2\sqrt{ab}$, the parameters should be chosen as $g = 2\gamma/(\gamma + 2\sqrt{ab})$ and $\Delta = (\sqrt{a} + \sqrt{b} - g)(\sqrt{a} - \sqrt{b})/(1-g)$, both of which lie in $[0,1]$. When ρ_{BD}^0 is entangled with $\gamma > 2\sqrt{ab}$, the probability distribution (30) can be decomposed as

$$P_Q(\rho_{BD}^0) = [1 - \mathcal{C}(\rho_{BD}^0)]P_L(\rho_{BD}^0) + \mathcal{C}(\rho_{BD}^0)P_{NL}(\rho_{BD}^0), \quad (33)$$

where $P_L(\rho_{BD}^0)$ takes the definition in Eq. (31) with $\vartheta = \arcsin[(\sqrt{a}-\sqrt{b})/(\sqrt{a}+\sqrt{b})]$ and $P_{NL}(\rho_{BD}^0) = \frac{1}{4}(1 + \mathbf{A}\cdot\mathbf{B}')$ corresponding to the probability distribution of the Bell state $|\psi^+\rangle$.

(ii) An arbitrary state (29) can always be written as $\rho_{BD} = x|00\rangle\langle 00| + y|11\rangle\langle 11| + p\rho'$, where $p = \gamma + a + b$ and $\rho' = \rho_{BD}^0|_{(\gamma,a,b)\to(\gamma/p,a/p,b/p)}$. Its concurrence is $\mathcal{C}(\rho_{BD}) = p\mathcal{C}(\rho')$. One can obtain immediately

$$P_Q(\rho_{BD}) = pP_Q(\rho') + xF^+(A_z)F^+(B_z) + yF^-(A_z)F^-(B_z). \quad (34)$$

In the approach given in step (i), $P_Q(\rho')$ can be divided into the local $P_L(\rho')$ and nonlocal $P_{NL}(\rho')$ parts, with the wights $1 - \mathcal{C}(\rho')$ and $\mathcal{C}(\rho')$ respectively. Choosing the construction $P_L(\rho_{BD}) = [p(1-\mathcal{C}(\rho'))P_L(\rho') + xF^+(A_z)F^+(B_z) + yF^-(A_z)F^-(B_z)]/[p(1-\mathcal{C}(\rho')) + x + y]$, one has

$$P_Q(\rho_{BD}) = [1 - \mathcal{C}(\rho_{BD})]P_L(\rho_{BD}) + \mathcal{C}(\rho_{BD})P_{NL}(\rho') \quad (35)$$

in which the nonlocal probability distribution is the same as the one of ρ'.

III. CONCLUSION

In conclusion, we investigate the EPR2 decomposition of the probability distribution arising from single-copy von Neumann measurements on arbitrary two-qubit states. In our constructive proof, the local content is shown to has a lower bound connected with the concurrence which measures the degree of entanglement, $p_L^{\max} \geq 1 - \mathcal{C}(\rho)$. The local probability distribution for two families of mixed states are constructed independent of the scheme in the proof. Both of them lead to the local weight $p_L = 1 - \mathcal{C}(\rho)$.

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